A MODEL OF SMITHIAN GROWTH AND INTERCONTINENTAL TRADE PROFITS IN EARLY MODERN EUROPE

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Abstract

This paper models how intercontinental trade profits could encourage growth in Early Modern Europe. Households produce and consume autarkic and market goods in an archipelago-like setting. A single trader monopolizes trade between them. He can accumulate capital to increase his trade capacities. This yields a gradual Smithian growth model with properties similar to a Cass-Koopmans model. By offering high profits, intercontinental trade encourages capital accumulation and growth. The predictions of the model are consistent with the growth experience of England, France and the Netherlands in the 17th and 18th century.

Keywords: Smithian growth, Early Modern Europe, Intercontinental trade, Growth model

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Introduction

Recent empirical work has shown that the size of intercontinental trade, including slave trade and trade in slave-produced colonial commodities had a positive effect on economic

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growth in Early Modern Europe (Allen 2003, Acemoglu, Johnson, and Robinson 2005). Acemoglu and his co-authors suggest as an explanation that the development of Atlantic trade reinforced the position of traders, who were thus able to coerce national governments in setting up institutions that defended property rights. However, protecting property rights was only a small part of what Early Modern states could do for traders (Hirsch 1991). Because of the evolution of both economic thought and internal political struggle, England, France and the Netherlands implemented international policies partly devoted to supporting the activity of domestic traders in the world economy, even if they were dealing in goods neither produced nor consumed by their own economy. They did that in a number of ways, ranging from direct subsidies to military action against competitors, in a specific European tradition started by Venice.[1] These activities lead domestic trader to enjoy a higher rate of profits in intercontinental trade in domestic activities, even when risk is taken into account.[2] Intercontinental trade profits could be maintained at a high level for European traders both because the extension of individual countries trade was often done at the expense of other countries and because European trade represented only a small part of world trade, especially in Asia. While it may be the case that other forms of trade provided high profits, intercontinental trade is the only one for which this argument can be made on a sound empirical basis.

The traditional explanation for the importance of intercontinental trade was that its high profits had an important role in Early Modern accumulation of capital. The strongest form of

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Garrec, Sandrine Levasseur, participants in the OFCE internal seminar, HEC Lausanne Brownbag seminar, and Oxford Graduate Workshop in Economic and Social History.

[1] Curtin 1984, p. 116.

[2] On the difficulties of computing intercontinental trade profits, see Daudin 2002a. On the profits in France, see Daudin 2004.

this idea was that this accumulation was at the root of the Industrial Revolution.[3] It is now discredited. A weaker form of this idea was that slave trade and plantation colonies played an important role in accumulation before the Industrial Revolution. This is still debated. Many economic historians would agree with O'Brien's view that profits from the "periphery," or, approximately, the non-European world, were simply too small to have played a major role in European growth even before the Industrial Revolution (O'Brien 1982, Eltis and Engerman 2000). In the case of France, it has recently been computed that savings from intercontinental trade had increased French GDP by as little as 2 to 3 % during the 18th century (Daudin 2006a and Daudin 2006b)[4]. Furthermore, economic logic does not support the view that investor would take out capital from a high-profit sector to supply the rest of the economy with capital…

This paper suggests a new explanation on why intercontinental trade might have played an important role in Europe's domestic capital accumulation before the Industrial Revolution. The intuition is simple and can be implemented in a basic model of economic growth inspired from multi-sectoral "AK" endogenous growth models as presented by Rebelo and studied, for example, by Glachant[5]. The idea of these "heart of growth" models is that even a small economic sector can play a decisive role in accumulation by offering a way to escape declining returns to capital. The suggestion of this paper is that intercontinental trade profits played a role through the encouragement of accumulation rather than through direct contribution to the capital stock.

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[3] This idea as been defended by Marxists and World-System historians: Williams 1944 (1966), Amin 1974; Frank 1978; Wallerstein 1989.

[4] Other potential roles for intercontinental trade have been suggested: e.g. its role in the development of financial markets or in breaking the Malthusian barrier… Inikori 1990, Pomeranz 2000.

[5] Rebelo 1991, Glachant 1995.

To show this obviously implies to build a plausible model of early modern growth that gives a role to capital. Recent developments in historical growth theory — most notably the literature on "unified growth theory" (Kremer 1993, Galor and Weil 2000, Hansen and Prescott 2002) do give a role to capital and could be used. But they are mainly concerned with the transition from a pre-modern economy with little growth per capita (associated with Malthus) to the post-industrial revolution economy of sustained growth per capita (associated with Solow). They do not study the logics of pre-modern growth. Furthermore, capital in the unified growth theory is mainly productive capital. However, it is well known that the intercontinental trading entrepreneurs and investors were not especially linked with industrial entrepreneurs and investors (e.g. Engerman 1972, Bairoch 1973, Devine 1976). It is more plausible to defend the idea that high returns in intercontinental trade encouraged capital accumulation among domestic traders than among the industrialists using productive capital.

This paper builds a model based an the old suggestion by historians that early modern economies were able to grow through Smithian mechanisms of deepening market integration (e.g. Jones 1998 and Mokyr 1990, p. 5). Circulating and trade capital accumulation by traders allowed them to extend their activity. They played two roles in the deepening market integration. First, they offered new consumption goods, which diffusion can be seen in probate inventories (Roche 1997, Baulant 1975 and Baulant 1989). Second, they had an active role in the organization of production and in offering outlets for market production, as suggested by the literature on proto-industry (starting with Mendels 1972, e.g. Kriedte, Medick, and Schlumbohm 1977 (1981)).

As mathematical Smithian growth models (for a review, see Yang and Ng 1998) have not yet been used to explore the economics of Early Modern Europe, this paper does so by offering a simple model of Smithian growth. This model is grounded in the idea of "industrious revolution", not as a substitute or an explanation for Industrial revolution, but as a mechanism for explaining some growth episodes in Early Modern Europe. The germ of this idea can be found in Smith's "vent for surplus" theory of international trade ({Oulton, 1993 #3437}). According to this view, one of the reasons for growth was the integration of households in the domestic market economy through proto-industry and market agriculture (de Vries 1994). It manifested, for example, through the increase in the number of work hours (Voth 1996). This model of Smithian growth is not based on transport and network externalities and yields different results from, for example, the Smithian model of Sung Chinese growth by Morgan Kelly (Kelly 1997). It forms an useful base to examine the effect of high profit from intercontinental trade.

The outline of the paper is as follows. In the following section, the paper develops a model of the European domestic economies before the Industrial Revolution. In section 3, the paper shows how this model is modified by the introduction of intercontinental trade as a high-profit sector. Section 4 confronts the predictions of the model with macroeconomic data on the Netherlands, England and France in the 17th and 18th century. Section 5 concludes.

1. A domestic Smithian economy

This paper models economies of specialization as a productive advantage for market goods compared to autarkic goods. The exchange of market production goods for market consumption goods has a cost: this creates a trade-off between economies of specialization and trade cost. The motivation for the industrious revolution is the reduction of trade costs. This reduction is modelled as depending on the behaviour of a maximizing domestic trader.

1.1. Households

This model centres on the decisions of producers regarding their participation to the market rather than on their consumption / saving trade-off. This participation is modelled as a transfer of productive capacities from the production of autarkic goods in favour of the production of specialised market goods.

1.1.1. Markets and goods

The economy is an archipelago of *I* symmetrical local markets[6]. Empirically, according the central place theory by Christaller and Lösch, they can be identified with the influence area of fairs or market towns. Studies have shown that in France and England such areas had approximately a six-kilometre radius[7]. Their small size allowed anyone to walk to the market town, do business and be back within a day. Inside each local market, exchanges are free. Each local market can trade with other markets through a "national" market at a certain cost.

There are no firms in the model. The only agents in local markets are households. They are both consumers and producers, akin to farmers[8]. Their inter-temporal behaviour is not modelled. There are three types of goods in the economy: Z-goods (autarkic), Y-goods (market production) and C-goods (market consumption). Their characteristics are presented in Table 1.

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[6] *I* is used as a scaling factor, but the model is written in such a way that *I* plays no role in the dynamics or the equilibrium levels.

[7] Braudel 1979, t. 2, p. 33-37 & pp. 121-124; Everitt 1985; Braudel 1986, vol. 1, p. 14; Margairaz 1988, pp. 31, 53 and 246, Thomas 1993, pp. 55-101.

[8] The model builds on the study of rural households: Hymer and Resnick 1969.

Table 1: Characteristics of goods

Goods	Number of varieties	Production in local markets	Consumption in local markets	Trade outside local markets	Examples
Z-goods	One	Yes	Yes	No	Subsistence agriculture and handicraft
Y-goods	One per local market (*I*)	Yes	No	Sold to the national market	Agricultural or industrial market goods: textiles, wine, furniture, hardware, etc.
C-goods	One	No	Yes	Bought from the national market	Consumption basket of different market agricultural and industrial goods

C-goods can be thought as baskets of Y-goods that have been bundled on the national market. A piece of cloth produced by a weaver is a Y-good. The bundle of goods he consumes – some of the same cloth along with other textiles, hardware, wine… – is a C-good. There is an overlap in the categories of goods included in Z-goods and in C/Y-goods: clothing, furniture, food products are present in both types of goods. The distinction is between high-quality or further processed goods that were sold on a larger market and mundane quality goods that were produced for local consumption by artisans and peasants (for the example of wheat, see Grantham 1989, p. 188 and Meuvret 1977).

Selling Y-goods and buying C-goods can only be done on the national market. The relative importance of C/Y-goods and Z-goods in consumption and production is a measure of market integration.

1.1.2. *Representative households*

Usual rules of perfect competition apply inside local markets. One can simply examine the behaviour of a representative household. Each representative household i has an equal production capacity that is written as:

$$y_i = Y\left(z_i\right) \quad (1)$$

Where y_i is the production of specific Y-goods and z_i the production of generic Z-goods. Y is strictly decreasing in z_i. GDP is equal to the sum of the production of Y-goods and Z-goods. Y is such that GDP increases when the production of Z-goods decreases, i.e. GDP increases when market integration increases.

1.1.3. *Households and the market*

As the Y-goods cannot be consumed or hoarded, each representative household sells its whole production of Y-goods (y_i) to the national market. It buys a c_i of C-goods. The budget equation of each household is:

$$p_{i,Y}y_i = p_C c_i \tag{2}$$

Where p_C is the price of C-goods and $p_{i,Y}$ the price of Y-goods on the national market. $p_{i,Y}$ is always smaller than p_C.

We can define a mark-up μ_i, varying between zero and one.

$$\mu_i = 1 - \frac{p_{i,Y}}{p_C} = \frac{y_i - c_i}{y_i} \tag{3}$$

This mark-up is a measure of the costs of participating to the market for households. If it is equal to zero, the household can exchange Y-goods for C-goods on a one-to-one basis. If it is equal to one, the household cannot get any C-goods whatever is its offer of Y-goods.

Neither Z-goods nor C-goods can be hoarded. They have to be consumed immediately. Each representative household consumes all its production of Z-goods and the quantity of C-goods it buys. Its utility is:

$$u_i = U\left(z_i, c_i\right) \tag{4}$$

The autarkic good is an inferior good.

1.1.4. *Household's choice*

In this setting, each household i chooses its optimal level of production y_i^* by solving the following program:

$$\begin{cases} \underset{y_i}{Max}\, U\left(z_i, c_i\right) \\ y_i = Y\left(z_i\right) \\ c_i = \left(1-\mu_i\right) y_i \end{cases} \quad (5)$$

If there are multiple solutions, households select the smallest Y-goods production possible. Hence y_i^* is unique. It can be written as a function R of μ_i:

$$y_i^* = R\left(\mu_i\right) \quad (6)$$

Because the autarkic good is inferior, R is decreasing in μ_i: both the substitution effect and the income effect encourage households to increase their participation to the market when the relative price of market consumption goods declines.

Y-goods production by households reaches zero for $\mu^{max} \leq 1$. R is strictly decreasing in the domain $[0, \mu^{max}]$. R^{-1} is defined from R restricted to that domain. I assume that $R'' < 0$ in that domain.

R plays a very important role in the model. The higher the relative price of market production goods (Y-goods) relative to the price of market consumption goods (C-goods), the more households contribute to the national market. This increases GDP and is at the core of the mechanism of growth this model studies.

1.1.5. *Application to a specific functional form*

To get tractable results, one needs to specify Y and U. The symmetry of local markets allows to drop the i subscript.

Y is a simple linear trade-off function.

$$y = A\left(Z - z\right) \quad (7)$$

Where A is a set of techniques and Z is the maximum level of Z-goods production. Both are scalars, and A is strictly superior to one. The model makes the extreme Malthusian assumption that production capacities are strictly limited by natural resources availability and do not depend on the population in local markets.

U is a simple separable utility function in which only Z-goods have a decreasing marginal utility:

$$U(z,c) = \frac{1}{N}\left(B.\ln(z) + c\right) \text{ with } 0 < B < A.Z \tag{8}$$

Where N is the size of the household and B is a parameter that measures the desirability of Z-goods compared to C-goods.

The program of the household can be written as:

$$\underset{y}{Max}\, U\left(Z - \frac{y}{A_i}, (1-\mu)y\right) \Leftrightarrow \underset{y_i}{Max} \frac{1}{N}\left(B.\ln\left(Z - \frac{y}{A}\right) + y.(1-\mu)\right) \tag{9}$$

If y^* is an interior solution of the household's program, it verifies:

$$\frac{dU}{dy}\left(y^*\right) = 0 \Leftrightarrow y^* = A.Z - \frac{B}{1-\mu} \tag{10}$$

Hence R can be defined as:

$$\begin{cases} \text{if } \mu < 1 - \dfrac{B}{AZ} \Rightarrow R(\mu) = A.Z - \dfrac{B}{1-\mu} \\ \text{if } \mu \geq 1 - \dfrac{B}{AZ} \Rightarrow R(\mu) = 0 \end{cases} \tag{11}$$

As expected, $R'(\mu)$ and $R''(\mu)$ are strictly negative for $\mu < 1 - \frac{B}{AZ}$.

1.2. Domestic Trade

Market participation costs are endogenized by studying the activities of domestic traders. Traders had to insure the logistic and marketing transaction costs (Coase 1937). Some are *ex* *ante* costs: finding information on the market in general and finding a particular exchange partner. Some are "instantaneous" costs: determining the goods to be exchanged, bargaining their price and the precise contract. Some are *ex post* costs: the mutual monitoring of exchange partners to insure the spirit and letter of a contract is respected by preventing late payment or delivery and preventing deceit on the quality of goods (Casson 1987 and Furubotn and Richter 2000, p. 44-45). The level of costs depended on the institutional framework. The means traders could use to pay these costs were numerous: information on markets in the form of human capital, exchanging bonds to prevent misbehaviour — hence accumulating "social capital", financial capital, etc… For the benefit of this paper, all this will be summed up as "trade capital" (for a discussion and a development of this concept, see Daudin 2005 and Daudin 2002b).

1.2.1. *Traders and trade function*

Traders are the only agents that can trade with every local market. They buy all the Y-goods produced by households and sell them C-goods. For simplification, traders are modelled as being represented by a single monopolist. Assuming Cournot-competition or Bertrand-competition with capacity constraints, competition between traders yields similar results (Daudin 2005, pp. 494-497).

The trader is infinitely lived. At each period, he consumes $I.c_{M,t}$ units of C-goods.

The trader has a constant inter-temporal elasticity function:

$$U_M\left(I.c_{M,t}\right)=\frac{c_{M,t}^{(1-\theta)}-1}{1-\theta}\text{ with }\theta>0\text{ and }\theta\neq 1 \tag{12}$$

His inter-temporal utility function is:

$$\sum_{t=1}^{t=+\infty}\frac{1}{\left(1+\rho\right)^{t-1}}U_M\left(I.c_{M,t}\right) \tag{13}$$

Where ρ is his preference for the present.

In the same way a production function defines the activity of a firm, a trade function defines the activity of a trader. Trade capital is an important input to that function. It is saved from C-goods, on a *I*-to-one basis. The trader holds at each period *t* a quantity of trade capital k_t. He can keep capital from period to period. The trader uses trade capital to transform Y-goods into C-good according to a "trade function" T_t. This function is akin to a production function, but with the important difference that trade capital cannot physically produce any new goods. Hence, in the trade function, trade capital and Y-goods inputs are strict complements[9]. This is very different from the usual "iceberg" trade costs.

$T_t^k(k)$ is the maximum amount of C-good that can be traded with *k* units of trade capital. This changes through time. This function T_k has the usual characteristics of a production function: $T_t^{k\prime} > 0$ and $T_t^{k\prime\prime} \leq 0$. As all local markets are symmetrical, to "produce" one unit of the C-good, the trader needs *1/I* unit of every Y-goods. Assuming that there are y_i inputs of each Y-goods and a quantity of capital *k* and dropping the time subscript, that restricts the form of the trade function to the following:

$$T\left(y_1, y_2, .., y_I, k\right) = Min\left[I.y_1, I.y_2, .., I.y_I, Y^k\left(k\right)\right] \tag{14}$$

1.2.2. Chronology of decisions

The economy goes through discrete time periods. At each period *t*:

[9] One way of thinking about the trade function, if one accepts nominal rigidities, is to assimilate trade capital to money and the trade function to a cash-in-advance constraint. In that case, "savings" represent exports of C-goods in exchange for money. However, this neglects the specific nature of social capital. This paper does not deal with these issues and treats the trade function as a black box.

- The trader chooses a mark-up μ_t and announces it to the households. Each household produces y_t Y-goods of its particular variety. The trader chooses μ_t so that he has enough capital to trade all Y-goods.

- He gathers all the Y-goods produced by households.

- He transforms them into $I.y_t$ C-goods with his trade function.

- He gives to the households as a group a share $(1-\mu_t)$ of the total C-goods available.

- He consumes $I.c_{M,t}$, taken either from his share of the C-goods produced in that period or from his stock of capital (on a one-to-I basis).

- He saves the rest and transforms it into capital that can be used in the following period: $k_{t+1} = k_t + \frac{\mu_t I.y_t - I.c_{M,t}}{I}$.

As the trader has full market power, he simply collects all Y-goods and then provides producers with C-goods according to the mark-up he has announced. That assumes that his commitment to μ is credible. Prices for produced goods and consumed goods were observable by households at the same time, and both nominal rigidities and competition between traders insured that price changes between before the production decision and after were not too large[10].

1.2.3. Trader's instantaneous choice

At each period, the trader knows the reaction function R_t of the households, who choose their production level according to the mark-up μ_t:

$$I.y_t = Min\left[I.R_t\left(\mu_t\right), I.R_t\left(\mu_t\right), .., I.R_t\left(\mu_t\right), T_t^k\left(k_t\right)\right] = Min\left[I.R_t\left(\mu_t\right), T_t^k\left(k_t\right)\right] \quad (15)$$

[10] Such prices changes happened, of course. For example a subsistence crisis could dramatically increase the prices of grains. This model neglects short-term market perturbations.

The strategic variable of the trader is the mark-up μ_t. The trader uses it to maximise the share of C-goods he keeps for himself: $\mu_t.I.y_t$. If there is no capital constraint, this is the same as maximising $\mu_t R(\mu_t)$. Let μ_t^* be the level of the mark-up that does that. It verifies:

$$\frac{d\left(\mu_t^* R_t\left(\mu_t^*\right)\right)}{d\mu_t^*} = 0 \Leftrightarrow R_t'\left(\mu_t^*\right)\mu_t^* + R_t\left(\mu_t^*\right) = 0 \tag{16}$$

To trade all the Y-goods implied by this optimal – for him – mark-up, the trader needs a quantity of capital equal to k_t^* defined as:

$$k_t^* = \left(T_t^k\right)^{-1}\left(R_t\left(\mu_t^*\right)\right) \tag{17}$$

If he does not have enough capital, he increases the mark-up so as to reduce the production of Y-goods down to a manageable level while increasing the share of C-goods he keeps for himself.

If R_t is strictly decreasing between μ_t^* and 1, it is possible to define a function P_t that gives the amount of C-goods the trader keeps from himself when he uses a certain quantity of capital in domestic trade. Dropping time subscripts, this function is defined as:

$$\begin{cases} \bullet\text{if } \ k < k^* \ : \ P(k) = \mu R(\mu) \text{ such as } \mu = R^{-1}\left(T^k(k)\right) \\ \text{i.e. } P(k) = R^{-1}\left(T^k(k)\right).T^k(k) \\ \bullet\text{if } \ k \geq k^*: P(k) = \mu^* R\left(\mu^*\right) \end{cases} \tag{18}$$

Proposition 1: P' ≤ 0 and P" ≥ 0.

Proof in appendix.

P's characteristics make it similar to a standard production function, except it is bounded.

It is also possible to write the relation between GDP per canton and the amount of trade capital used by the trader in domestic trade:

$$GDP(k)=\frac{T^k(k)}{I}+Y^{-1}\left(Z,\frac{T^k(k)}{I}\right) \tag{19}$$

1.2.4. Dynamic optimisation

As the number of local markets does not make any difference in the model, I is normalized to 1 in what follows.

The trader's program can be written as:

$$\begin{cases} \underset{\mu^1\ldots;k^1\ldots}{\text{Max}} \sum_{t=1}^{t=+\infty} \frac{1}{(1+\rho)^{t-1}} U_M\left(c_{M,t}\right) \\ k_{t+1}=k_t+P_t\left(k_t\right)-c_{M,t} \\ k_1 \text{ fixed} \end{cases} \tag{20}$$

The evolution of P_t through time depends on the techniques for trade and production and on the population in local markets. The assumption that P_t is constant through time, while probably not realist, allows us to study how the introduction of high-profit intercontinental growth changes the growth regime[11]. In that case, the trader's program is similar to the canonical Cass-Koopmans model ({Cass, 1965 #3401}, {Koopmans, 1965 #3402} and (Barro 1995, chapter 2)).

The dynamic has a saddle-point equilibrium, toward which the capital stock will converge due to the transversality condition. If the trader has a smaller capital stock than the optimum, he will gradually accumulate capital up to a fixed point determined by the equality of his rate of preference for the present and the marginal productivity of trade capital. If he

[11] Considering our assumptions, we could not find any functional form for the household utility, its production function and the trading function that would allow to write $P_t(k_t)$ in an non-trivial $I(t)P(k_t)$ form that would allow to conduct the same study because $I(t)$ would be equivalent to neutral technical progress.

does have a non-null preference for present, this fixed point is smaller than the optimal mark-up k^*.

Proposition 2: The stock of capital will converge toward a fixed point k^f that verifies:

$$\rho = P'\left(k^f\right) \tag{21}$$

This is a well known result. Proof can be found in the references. The fact that P is bounded is not an issue as production is also bounded in the Cass-Koopmans model.

It does not depend on the parameters of U_M. Depending on parameters of P, this fixed point might exist or not. If it does not, the trader is better off consuming all his capital in the first period.

1.2.5. Application to a specific functional form

These results can be verified for the household's utility function selected in equation (8). In that case, $\mu_t R\left(\mu_t\right)$ and μ^* can be written explicitly:

$$\begin{aligned}
&\mu R\left(\mu\right) = \mu \frac{B + A.Z\left(\mu - 1\right)}{\mu - 1} \\
&\frac{\partial\left(\mu R\left(\mu\right)\right)}{\partial\mu} = \frac{-B + A.Z\left(1 + B\right)\left(\mu - 1\right)^2}{\left(\mu - 1\right)^2} \\
&\frac{\partial\left(\mu^* R\left(\mu^*\right)\right)}{\partial\mu} = 0 \Rightarrow \mu^* = 1 - \sqrt{\frac{B}{A.Z}}
\end{aligned} \tag{22}$$

(The other solution is excluded by $\mu < 1$)

With the further assumption that $T_k(k)=T.I.k$ (T being a measure of the state of the transaction technology and institutions), it possible to write k^*:

$$k^* = \frac{1}{T}\sqrt{AZ}\left(\sqrt{AZ} - B\right) \tag{23}$$

It is also possible to write $P(k)$:

$$\begin{cases} \bullet if\ k < k^*,\ P(k) = k.T.\left(1 - \dfrac{B}{AZ - k.T}\right) \\ \bullet if\ k \geq k^*,\ P(k) = \left(\sqrt{AZ} - B\right)^2 \end{cases} \tag{24}$$

One can verify that $P'>0$ and $P''<0$ if $k < k^*$.

And:

$$GDP(k) = k.T + Z - \left(\frac{k.T}{A}\right) \tag{25}$$

One has to assume that A, Z and T do not change through time to use proposition 2. In that case, k^f exists if:

$$P'(0) > \rho \Leftrightarrow \frac{T\left(AZ - B\right)}{AZ} > \rho \tag{26}$$

If it does, it is possible to write k^f and the associated GDP:

$$\begin{aligned} k^f &= \frac{A.T.Z(T-\rho) - \sqrt{A.B.T.Z.(T-\rho)}}{T(T-\rho)} \\ GDP\left(k^f\right) &= Z + \frac{(A-1)\left(A.Z.(T-\rho) - \sqrt{A.B.T.Z.(T-\rho)}\right)}{A.(T-\rho)} \end{aligned} \tag{27}$$

As expected, the level of the GDP at the fixed point is increasing with *A* and *T*.

1.2.6. Consequences of technical and institutional progress

Solow growth was not the exclusive force of growth in Early Modern Europe. New technologies were invented and put in use. New institutional settings made trading easier as legislation better protected property rights or became more supportive of trading activity. In the framework offered by this model, these would translate respectively in a change in *Y* and in T_k.

The level of market production is limited by the interplay between the level of trading capital and T_k. An favourable change in Y that would encourage the household to produce and consume more market goods at a given mark-up would not influence it in the short term: it would simply cause an increase the mark-up μ and in Z-goods production. However, it also increases the optimal mark-up μ^* and the long-run level of Y-goods production and GDP.

In the short-term, a favourable change in T_k increases Y-goods production and reduces the mark-up μ because it is similar to an "advance" of the trader on his accumulation path. Yet, a change in T_k has no effect on the optimal mark-up or the optimal level of production for the trader. It simply changes the amount of trading capital necessary for this optimal level. Because it eases the constraints on accumulation, it will narrow the gap between the fixed-point of the trader's accumulation of capital k^f and the optimum stock of capital k^*. Hence it increases the level of Y-goods production and GDP in the long-run.

Technical progress has little short-term effect on Y-goods production, but increases its long-run level by increasing both the optimal level of trade capital and the fixed point of the trader's accumulation of capital. Institutional change has an important short-term effect on Y-goods production and changes its long-run level by narrowing the gap between the fixed point and the optimal level of capital.

2. <u>The role of international entrepot trade in Smithian growth</u>

In the preceding section, this paper has presented a gradual mechanism describing Smithian growth by the accumulation of transaction means. What is the effect of introducing intercontinental trade as a a high-profit sector in this mechanism?

2.1. Setting up the model

To come back to the initial argument, a number of writers have been impressed by the rate of profit available to individuals in intercontinental trade. In this paper, this sector is defined by its rate of profit and it provides a constant returns to trade capital equal to r, with $r > \rho$.

Furthermore, the model also assumes that a specific consumption good can be provided by the rest of the world to domestic traders. One unit of this consumption good can be bought in exchange for *1/I* unit of trade capital. This consumption good is not perfectly substitutable with domestic goods in the trader's utility function. Goods are associated in a Cobb-Douglas way in the trader's utility function. Let $I.x_t$ be the trader's consumption of these goods in period t. The trader's new instantaneous utility function is:

$$U_M\left(I.c_{M,t}, I.x_t\right) = \frac{\left(c_{M,t}^{\alpha}.x_t^{1-\alpha}\right)^{(1-\theta)} - 1}{1-\theta} \text{ with } \theta > 0, \theta \neq 1 \text{ and } 0 < \alpha < 1 \tag{28}$$

The quantity of trade capital invested in intercontinental trade at period t is called $k_{x,t}$. The quantity of trade capital invested in domestic trade at the same period is called $k_{d,t}$. I is normalized to 1.

The trader's program becomes:

$$\begin{cases} \underset{\mu_1 \ldots; k_1 \ldots}{Max} \sum_{t=1}^{t=+\infty} \frac{1}{(1+\rho)^{t-1}} U_M\left(c_{M,t}, x_t\right) \\ c_{M,t} \leq P\left(k_{d,t}\right) \\ k_t = k_{d,t} + k_{x,t} \\ k_{t+1} = k_t + P\left(k_{d,t}\right) - c_{M,t} - x_t + r.k_{x,t} \\ k_{x,t}, k_{d,t} \geq 0 ; \\ k_1 \ fixed \end{cases} \Leftrightarrow \begin{cases} \underset{\mu_1 \ldots; k_1 \ldots}{Max} \sum_{1}^{+\infty} \frac{1}{(1+\rho)^{t-1}} U_M\left(c_{M,t}\right) \\ P\left(k_{d,t}\right) - c_{M,t} \geq 0 \\ k_{t+1} = k_t + P\left(k_{d,t}\right) - c_{M,t} - x_t + r\left(k_t - k_{d,t}\right) \\ k_t \geq k_{d,t} ; k_{d,t} \geq 0 \\ k_1 \ fixed \end{cases} \tag{29}$$

There are three control variables ($k_{d,t}$, $c_{M,t}$ and x_t) and a state variable (k_t).

To simplify notations, the "M" subscript is dropped.

The associated Lagrangian is:

$$\begin{aligned} L = \sum_{1}^{+\infty} \frac{1}{(1+\rho)^{t-1}} \Big[U(c_t, x_t) + \lambda_t \big(k_t + P(k_{d,t}) - c_t - x_t + r(k_t - k_{d,t}) - k_{t+1} \big) \\ + \gamma_t \big(k_t - k_{d,t} \big) + \beta_t \big(P(k_{d,t}) - c_t \big) \Big] \end{aligned} \tag{30}$$

First order conditions are:

$$\begin{aligned} \frac{\partial L}{\partial c_t} = 0 \quad &\Leftrightarrow \lambda_t = U_1'(c_t, x_t) - \beta_t \\ \frac{\partial L}{\partial x_t} = 0 \quad &\Leftrightarrow \lambda_t = U_2'(c_t, x_t) \\ \frac{\partial L}{\partial k_{d,t}} = 0 \quad &\Leftrightarrow \lambda_t \big[P'(k_{d,t}) - r \big] - \gamma_t + \beta_t P'(k_{d,t}) = 0 \\ &\Leftrightarrow \lambda_t = \frac{1}{r} \big(U_1'(c_t, x_t) P'(k_{d,t}) - \gamma_t \big) \\ \frac{\partial L}{\partial k_t} = 0 \quad &\Leftrightarrow \lambda_t = \frac{(1+\rho)\lambda_{t-1} - \gamma_t}{1+r} \end{aligned} \tag{31}$$

The transversality condition requires that the discounted capital stock valued at its shadow price converges toward 0. This can be written as:

$$\lim_{t \to +\infty} \frac{k_t \lambda_t}{(1+\rho)^{t-1}} = 0 \tag{32}$$

2.2. Dynamics

It is useful to define k^D:

$$k^D \text{ either verifies } P'(k^D) = r \text{ or is equal to 0 if } P'(k) < r \text{ for all } k \tag{33}$$

Proposition 3: The stock of domestic capital converges toward k^* as defined in equation (17)).

The dynamics of this model and the proof of this proposition are in the appendix. The intuition follows.

Suppose the trader starts with a capital stock sufficiently small that returns to capital in the domestic economy are higher than in intercontinental trade: $k_0 < k^D$. The dynamic starts in the first regime. The trader accumulates capital in the domestic economy only and gradually increases its consumption. His increase in consumption (given by the decrease of λ at a rate greater than $1-\frac{1+\rho}{1+r}$) is faster than if there was no possibility of investment in intercontinental trade. At some point, his stock of capital reaches k^D, bringing him to the second regime.

In the second regime, the returns to the domestic capital stock are equal to the returns to capital invested in intercontinental trade. The domestic trade capital stock stops growing and savings are channelled into intercontinental trade. In this regime, the domestic economy stagnates. But the trader's consumption and capital stock both increase. At some point, as λ decreases at a constant rate, his domestic consumption level reaches the maximum level possible with only k^D invested in domestic trade. The dynamic then moves to the third regime.

In the third regime, the limited substitutability between domestic and foreign goods for consumption is binding. As a result, the trader consumes all his domestic income. In order to keep on increasing his domestic consumption, the trader has to invest more capital in domestic trade than k^D, despite the lower returns compared to intercontinental trade,. His domestic capital is bounded by k^* which maximises his domestic income. As $k^* > k^f$, domestic GDP increases further in the long-run than in the model without intercontinental trade. Notice that, once his stock of domestic capital is close enough to its long-term value k^*, it can be considered as constant. With this approximation, the model takes the usual AK form, which dynamics are well known: the stock of capital invested in intercontinental trade grows without bound. Table 2 sums up the characteristics of each regime.

Table 2: Characteristics of the regimes

Regimes	Domestic trade capital	Intercontinental trade capital	Local consumption constraint	Returns to domestic capital
First	Accumulation, < k^D	No accumulation	Is not active	$> r$
Second	No accumulation, = k^D	Accumulation	Is not active	$= r$
Third	Accumulation $k^D < k < k^*$	Accumulation without bound	Is active	$< r$
Fourth	No accumulation	No accumulation	Is active	$\geq r$

These three regimes are a useful abstraction to understand the logics of the model. But, rather than the specific dynamics, the important result of this model is that intercontinental trade, through higher profits, pushes the "Industrious Revolution" further than in an economy without intercontinental trade investment.

Furthermore, the promises of riches to be made in intercontinental trade increases the pace of capital accumulation (For more details on the plausibility of these dynamics in the case of France, see Daudin 2006a). This is mitigated early in the dynamic by the role of intercontinental trade in capturing a share of the stock of the capital. In the long run, however, the domestic capital stock predicted by the model with intercontinental trade is greater than the domestic capital stock predicted by the model without intercontinental trade. This is linked to the fact that foreign goods cannot fully substitute for domestic goods in consumption: it encourages the trader to invest in domestic trade despite declining profits.

3. The European experience

To understand the scope of the effect of introducing intercontinental trade, one can attempt a calibration based on the experience of Europe.

3.1. Why is Europe interesting?

Europe is a good example for two reasons. First, as noted in the introduction, there is a reasonable case to be made about the importance of Smithian growth in Europe before the Industrial Revolution and the advent of techniques-driven Solow growth. Second, the model needs large states for the full advantages of intercontinental trade to exist. If domestic sources of consumption are not important enough – or foreign sources of consumption can substitute for them – capital accumulation will be encouraged, but the centres of international trade would become simply enclaves of growth consuming foreign goods. The states engaged in European Mercantilist power politics in the 17th and 18th century were large enough to avoid this difficulty.

Not all the European states would be good examples, but only those that had both the power and the willingness to help domestic traders. If they predated on domestic traders as well as on foreign traders, as it was the case in Spain, they discouraged accumulation. Furthermore, if they were not powerful enough to insure high profits to their traders, they could not benefit from the mechanisms presented in this model. That was the case of the Austrian Hapsburgs, who could not fully support the Ostend Company; of Portugal as it became subordinated to British trade interests after the treaty of Methuen; of most non-European states during that period and even of Spain in the 18th century, despite the reforms by the Bourbons.

These restrictions leave us with three potential case studies for the application of the model: England (associated with Wales in this paper), France and the Netherlands.

3.2. Collecting data

Data has been gathered around four years: 1655, 1716, 1753 and 1790[12]. There is no way to determine exact values for each of these years, but approximations of the mean value of each variable around each year can be constructed. Each year was chosen for both data availability and chronological reasons.

1655 is the earliest date for which there is reliable trade data for the Netherlands. Furthermore, it represents a turning point in European history: after solving internal problems, England's Navigation Acts of 1651 and the nomination of Colbert as France's *contrôleur general* in 1661 symbolize the will of both these countries to counter Dutch trade supremacy.

1716 is the earliest for which there is reliable trade data for France. The long-lasting European wars associated with Louis XIV just finished with the treaties of Utrecht (1713) and Rastatt (1714) and the following period was relatively peaceful – at least between the three countries under study – up to the War of Austrian Succession (1740-1748, direct conflict between France, Great-Britain and the Netherlands only started in 1744).

1753 is the least satisfying of our four dates, as it is simply between the War of Austrian Succession and the Seven Year War (1756-1762). Yet, it cuts the period between 1716 and 1790 neatly in two sub-periods of 37 years.

Finally, 1790 does not need much justification. Political and economic events in the late 18th and early 19th century marked the beginning of the domination of England on its economic and political rivals and the beginning of a new economic era.

Many current studies on Early modern Europe development are based on the examination of real wages rather than GDP (this is the case for example of Allen 2003). It is certain that estimates of real wages are of higher quality than estimates of GDP, especially since the recent new wage of price data collection (see Maddison 2003, pp. 251-3 for a discussion). However, Europe-wide comparable wages concern mainly urban unskilled workers. Most workers in cities were already integrated in the regional or national market economy at the beginning of the period. The Smithian growth of the market economy was mainly a rural phenomena. Furthermore, real wages are mainly computed relative to food prices. Food was certainly a very important budget item, but the "industrious revolution" is based on the diffusion of new consumer goods, and using real wages would miss the relative price decrease or the apparition of new consumption goods (de Vries 1992, Clark 2005). For these reasons, GDP is a better proxy of development. Data on real GDP and intercontinental trade estimated in silver were collected. There are no source that would allow to deflate trade in order to compute "real" trade — but that should not trouble the inter-country comparison. The details of data collection and price treatment are presented in the data appendix.

3.3. The numerical exercise

The functional forms used in the numerical exercise were the ones presented in the preceding section. Parameters values have been chosen according to Table 3.

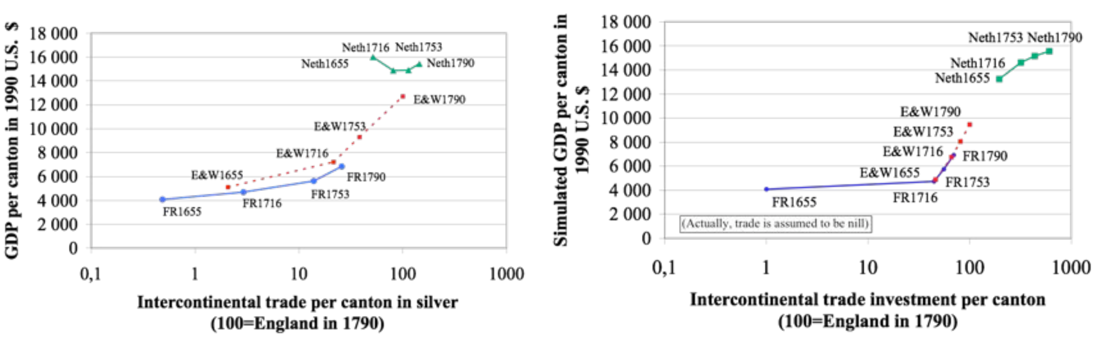
[12] It was not possible to find data on the "failed" cases of Spain and Portugal.

Table 3: Value of the parameters for the simulation

Z, the maximum level of production of autarkic goods per canton (in thousands of 1990 U.S. $)	**350**. Based on Maddison's data on GDP in 1000. He assumes the GDP per capita in Europe to be uniformly 400$. It comes down to a question of population density. As the surface of the Netherlands is not well know for this period and Southern France probably had some urban life (which is, by definition, non autarkic) I use England in 1000 as a benchmark, for a production per canton of 336,000 $
The minimum mark-up	**15 %**. This number was often used as a benchmark for the "just" trade profit in transaction, as in the Maximum French legislation (Le Roux 1996, p. 33).
A, the set of techniques that allow market goods production.	**A: 300 and B: 75,000**. As the Netherlands experienced very little growth from 1655 to the early 19th century, even though they had attained in 1655 of level of per capita GDP superior to what England would attain at the beginning of the Industrial Revolution, their level of development in 1655 is used as a benchmark for a fully commercialised economy with access to investment in intercontinental trade.
B, the parameter that determines the relative utility of autarkic goods compared to market goods.	
ρ, the trader's internal discount rate	**5%**. This is actually higher than some domestic interest rates. This is not in contradiction with the model: domestic interest rate declines to zero in the third regime whatever the traders' discount rate.
r, the rate of return of investment in intercontinental trade	**6%** (Daudin 2004)
T, the set of domestic exchange techniques	**0.259**: This assumes that France in 1655 was on the verge of developing its investment in intercontinental trade.
θ, elasticity of marginal utility (1/*θ* is the inter-temporal elasticity of substitution)	Difficult to estimate (See for example Beaudry and van Wincoop 1996). Fixed at **1.5**
α, the share of domestic goods in the trader's consumption	**90%**: Arbitrary, as there are no budget data for these social categories
Starting capital stocks	France and England: chosen so that 1655 GDP is right Netherlands: chosen so that 1790 GDP is right

Figure 1 compares the simulated and actual evolution of growth and intercontinental trade per canton (120 square kilometres abstract geographical units).

Figure 1: Actual and simulated Western Europe, 1655-1790



The simulation results are strongly constrained by the assumption that the only difference between England, France and the Netherlands were their starting capital stock. It is certain that the Netherlands and England had a "better" trade and production technology than France. Many phenomena were happening that are not modelled. The most important was the beginning of the Industrial Revolution in England during the second half of the eighteenth century. That certainly encouraged the growth of English GDP. Another is that the Netherlands were unable to extend their share of world trade anymore due to a small central state that could not muster domestic resources at a level comparable to the French or the English. This was due to military and political factors (O'Brien 2000), but also to the fact that the stagnation of their GDP did not give them the means to secure the very large levels of intercontinental trade predicted by the model.

This being said, the history of these three countries is broadly consistent with the model. For example, the model does track the correlation between the growth of English and French economies and the growth of their intercontinental trade. The data are compatible with a story in which England first and then France went into a catch-up phase by capturing a larger and larger share of world trade. That encouraged their domestic trade capital accumulation and Smithian growth. This process was quicker in England than in France, for reasons probably linked to the relative efficiency of each central state in securing trade investment opportunities by capturing a greater share of world trade.

The model explains only 65 per cent English growth and over-predicts French growth by 10 per cent. In the model, T has no effect on the dynamics in the third regime (even if it intervenes in the level of development). Hence, the growth difference cannot be attributed to institutional differences in the organisation of domestic trade. However, they could come from a difference in the respective efficiency of the English and French States in defending national traders in the international economy. It could be that, as a result, the proper profit rate on intercontinental trade for English traders is higher than in France. Putting it at 7 per cent for England allows the model to explain the whole of English growth. But it might be that the Industrial Revolution is a better explanation.

Without the investment possibility in external trade, the model predicts that the French and the English economy should not have been able to grow past 6,500,000 1990 US $ per canton. In the time frame, the French economy could grow to 6,000,000 1990 US $ and the English economy to 6,250,000 1990 US $. The introduction of intercontinental trade increases predicted French growth by 40 % and predicted English growth by 90%.

4. Conclusion

This paper has presented three new and complementary ideas. First, it has built a model of Smithian trade explaining the Industrious Revolution by the extension of trade capital. Second, it has shown that growth in this model could be boosted by the introduction of high-profit investment in intercontinental trade. Finally, it has confronted this model to the European experience of the 17th and 18th century. In accordance with the model, development of intercontinental trade was correlated with accelerated economic growth and increased long-term GDP, at least up to the ceiling that was reached by the Netherlands in the 17th century. This gives plausibility to the pertinence of the growth mechanisms highlighted in the model. Further confirmation could be given by the examination of the actual changes in prices and those predicted by the model.

However, the micro-economic foundations of the model were chosen for tractability reasons in order to demonstrate that intercontinental trade could have had an effect and to gauge its scope. They assume the absence of technological change and the absence of effect of population growth on production capacities. They lead to a macro-economic production function that is quite *sui generis* and is certainly not the best one to describe economy-wide relations. If one accepts the plausibility of the growth mechanisms in the mode, they could be integrated in different macroeconomic functions that might be more versatile to explain growth before the Industrial Revolution.

5. Mathematical Appendix

5.1. Proof of Proposition 1

Trivially, $P'(k) = P''(k) = 0$ if $k \geq k^*$.

Notice that, according to equation (16) and equation (17), k^* is the unique k that verifies:

$$R'\left(R^{-1}\left(T_k\left(k^*\right)\right)\right).R^{-1}\left(T_k\left(k^*\right)\right)+T_k\left(k^*\right)=0 \tag{34}$$

Because of this relation and $T' > 0$:

$$\begin{aligned}
&P'\left(k\right)=0\\
&\Leftrightarrow \frac{d\left(R^{-1}\left(T_k\left(k\right)\right).T_k\left(k\right)\right)}{dk}=0\\
&\Leftrightarrow T_k'\left(k\right).\left(R^{-1}\left(T_k\left(k\right)\right)+\frac{T_k\left(k\right)}{R'\circ R^{-1}\left(T_k\left(k\right)\right)}\right)=0\\
&\Leftrightarrow R'\circ R^{-1}\left(T_k\left(k\right)\right).R^{-1}\left(T_k\left(k\right)\right)+T_k\left(k\right)=0\\
&\Leftrightarrow k=k^*
\end{aligned} \tag{35}$$

As $P'(0) = T_k'\left(k\right).\mu^{\max}$ – which is superior to 0 – and P' is continuous, $P' > 0$ over the domain $[0,k^*[$.

For P'', we have assumed that R'' was < 0. Hence:

$$P''(k)=\left(T_k'(k).\left(R^{-1}\circ T_k(k)+\frac{T_k(k)}{R'\circ R^{-1}\circ T_k(k)}\right)\right)'$$

$$=\underbrace{T_k''(k)}_{-}.\underbrace{\left(R^{-1}\circ T_k(k)+\frac{T_k(k)}{R'\circ R^{-1}\circ T_k(k)}\right)}_{\text{Same sign as } P':+} \tag{36}$$

$$+\underbrace{T_k'(k)}_{+}\left(\begin{array}{l}\underbrace{\frac{T_k'(k)}{R'\circ R^{-1}\circ T_k(k)}}_{-}\\ \underbrace{-T(k).\frac{T_k'(k).\left(R^{-1}\right)'\circ T_k(k).R''\circ R^{-1}\circ T_k(k)}{\left(R'\circ R^{-1}\circ T_k(k)\right)^2}}_{-}\\ \underbrace{+\frac{T_k'(k)}{R'\circ R^{-1}\circ T_k(k)}}_{-}\end{array}\right)$$

Hence $P'' < 0$ over the domain $[0,k^*[$.

5.2. Proof of Proposition 3

5.2.1. *Regimes*

– First regime: $k_t=k_{d,t}$; $c_t < P(k_{d,t})$, $\gamma_t > 0$, $\beta_t = 0$. In this regime, there is no investment in intercontinental trade as the returns to capital are higher in the domestic economy.

– Second regime: $k_t > k_{d,t}$; $c_t < P(k_{d,t})$, $\gamma_t = 0$, $\beta_t = 0$. In this regime, the trader invests his capital in both intercontinental trade and the domestic economy. As the consumption of domestic goods is smaller than domestic production and this constraint is binding, the returns to capital is equal in both the domestic economy and intercontinental trade. Hence, the stock of domestic capital is not increasing.

– Third regime: $k_t>k_{d,t}$; $c_t = P(k_{d,t})$, $\gamma_t = 0$, $\beta_t > 0$. In this regime also, the trader invests his capital in both intercontinental trade and the domestic economy. But the consumption of domestic goods is now equal to domestic production. Because this constraint is binding, the trade-off between investment in the domestic economy and in intercontinental trade is akin to a trade-off between consumption and savings. The returns to capital in the domestic economy is smaller than in intercontinental trade. Both capital stock are increasing.

– Fourth regime: $k_t=k_{d,t}$; $c_{M,t}=P(k_{d,t})$, $\gamma_t>0$, $\beta_t>0$. In this regime, the whole capital is invested in the domestic economy and all of it is consumed. The dynamic path is trivial, as there is no capital accumulation.

5.2.2. Borders

We study the dynamic in the (k,λ) space.

The border between the first and the fourth regime is defined by: $c_{M,t}=P\left(k_{d,t}\right)$. That implies:

$$\begin{cases}\lambda_t = U_1'\left(P\left(k_t\right),x_t\right)\\ U_1'\left(P\left(k_t\right),x_t\right)=U_2'\left(P\left(k_t\right),x_t\right)\end{cases} \tag{37}$$

The border between the first and the second regime is defined by:

$$P'\left(k_t\right)>r \Rightarrow k_t<k^D \quad \left(k^D \,/\, P'\left(k^D\right)=r \text{ or } 0 \text{ if } P'(k)<r \text{ for all } k\right) \tag{38}$$

The border between the second and the third regime is defined by:

$$P\left(k_{d,t}\right)-c_t=0 \Leftrightarrow P\left(k^D\right)>c_t \Leftrightarrow \lambda_t>\lambda^D \tag{39}$$

Where λ^D is either equal to $+\infty$ if $k^D=0$ or defined such as:

$$\begin{cases}\lambda^D = U_1'\left(P\left(k^D\right),x_t\right)\\ \lambda^D = U_2'\left(P\left(k^D\right),x_t\right)\end{cases} \tag{40}$$

The border between the third and the fourth regime is defined as:

$$k_t = k_{d,t} \Leftrightarrow \begin{cases} U_2'\left(P\left(k_t\right), x_t\right) = U_2'\left(P\left(k_{d,t}\right), x_t\right) \\ \dfrac{U_1'\left(P\left(k_t\right), x_t\right) P'\left(k_t\right)}{r} = U_2'\left(P\left(k_t\right), x_t\right) \end{cases}$$
$$\Leftrightarrow \begin{cases} \lambda_t = U_2'\left(P\left(k_t\right), x_t\right) \\ \dfrac{U_1'\left(P\left(k_t\right), x_t\right) P'\left(k_t\right)}{r} = U_2'\left(P\left(k_t\right), x_t\right) \end{cases} \tag{41}$$

5.2.3. *Dynamics*

The dynamics of capital are defined as:

$$k_{t+1} = k_t + r\left(k_t - k_{t,d}\right) + P\left(k_{t,d}\right) - x_t - c_t$$

The dynamics of λ are defined in the first regime as:

$$\lambda_{t+1} = \frac{1+\rho}{1+P'\left(k_{t+1}\right)} \lambda_t \tag{42}$$

And in the second and third regime as:

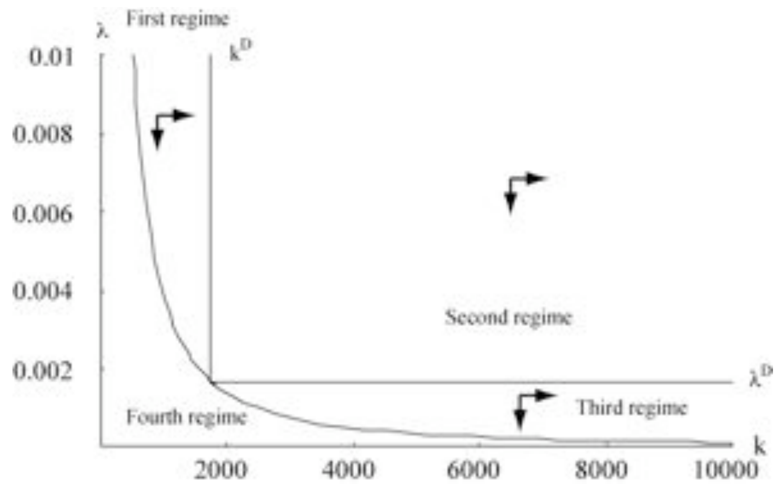
$$\lambda_{t+1} = \frac{1+\rho}{1+r} \lambda_t \tag{43}$$

The tranversality condition in the second and third regimes implies that the total capital stock cannot increase by more than *r* percent per period.

5.2.4. *Phase diagram*

Here is a possible phase diagram that corresponds to this model:[13]

[13] It is built from the same parameters as the paper's simulation, except that *r* is equal to 0.04 so that k^D and λ^D exist along with the first and second regime.



6. Data Appendix.

	England and Wales	Netherlands	France
Real GDP	Based on: Maddison 2001, p. 247 for 1600, 1700 and 1801. For 1655, I assume a constant growth rate between 1600 and 1700 For 1716 and 1753, I extrapolate from 1700 using the growth rate provided by Crafts 1985, p. 45 For 1790, I retropolate from 1801 using the growth rate in Crafts and Harley 1992, p. 715	I use the mid-range estimate of de Vries and van der Woude's estimate of the GDP per capita of the Netherlands at 1720-1744 prices: de Vries and Woude 1997, p. 707. I convert the unit from 1720-1744 guilders to 1990 international dollars by comparing their estimate for 1700 and Maddison's estimate for 1700: Maddison 2003, p. 46.	Based on Maddison 2003, p. 46 for 1700 and 1820. For 1655 and 176 I assume stagnation of real GDP income from 1655 to 1716 because of hunger crisis and warfare. This is Maddison's hypothesis based on Vauban and Boisguilbert: Maddison 2003, p. 20-22 and 27. For 1790, I retropolate from Maddison's number for 1820 using the growth rate from Toutain's growth rate of real GDP between 1785 and 1820 (Toutain 1997, p. 19) For 1753, I assume a constant growth rate between 1716 and 1790
Nominal trade – 1655: France and England	Léon estimates that in 1660 there were 3,000 Dutch trading ships, 600 French ones and slightly more English ones (here 650) (Léon and Carrière 1970 (1993), p. 187.). Dutch nominal trade in silver is known (see *infra*). English (and French) trade are assumed to be proportional to their number of trading ships. This yields a higher level of trade for England than if we retropolate King's numbers for 1688 with the growth rate of English trade from 1688 to 1700 as given by duty sources. I assume no re-exports from England and France.		
Nominal trade (All exchange rates to silver are taken at par from McCusker 1978, p. 9 and p. 95)	For 1716, 1753 and 1790: I use numbers from Davis and Deane & Cole for 1789-1790 (Davis 1969, p. 94, 102 and Deane and Cole 1962 (1969), p. 87) and assume constant growth rates between estimates to compute trade in early 18th century prices For 1784-6 numbers, I convert them from British trade to English trade by using the ratios based on 1772-3 numbers as given by Dean and Cole. Idem for re-exports. I then assume these data are in 1700 prices and use Officer 2001 to deflate them.	1655, 1716 and 1753: I use estimates for 1650s, 1720s and 1770s by de Vries and Woude 1997, p. 499 and assumes constant growth rates between estimtes. 1790: estimated by de Vries and Woude 1997, p. 495. I assume that the growth rate of the share of re-exports from the 1770s to 1790 is equal to the growth rate from the 1720s to the 1770s.	1716 and 1790: from Arnould 1791, table 2 (for 1790, extrapolated from 1787 numbers using the 1753-1787 growth rate). It provides re-exports as well (table 2-F) 1753: from Arnould, table 10, using as a modifier the mean between the ratio between his numbers and the Bureau's in 1790 and 1716: +168% (See Daudin 2005, p. 201). Re-export share extrapolated from 1716 and 1790.

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